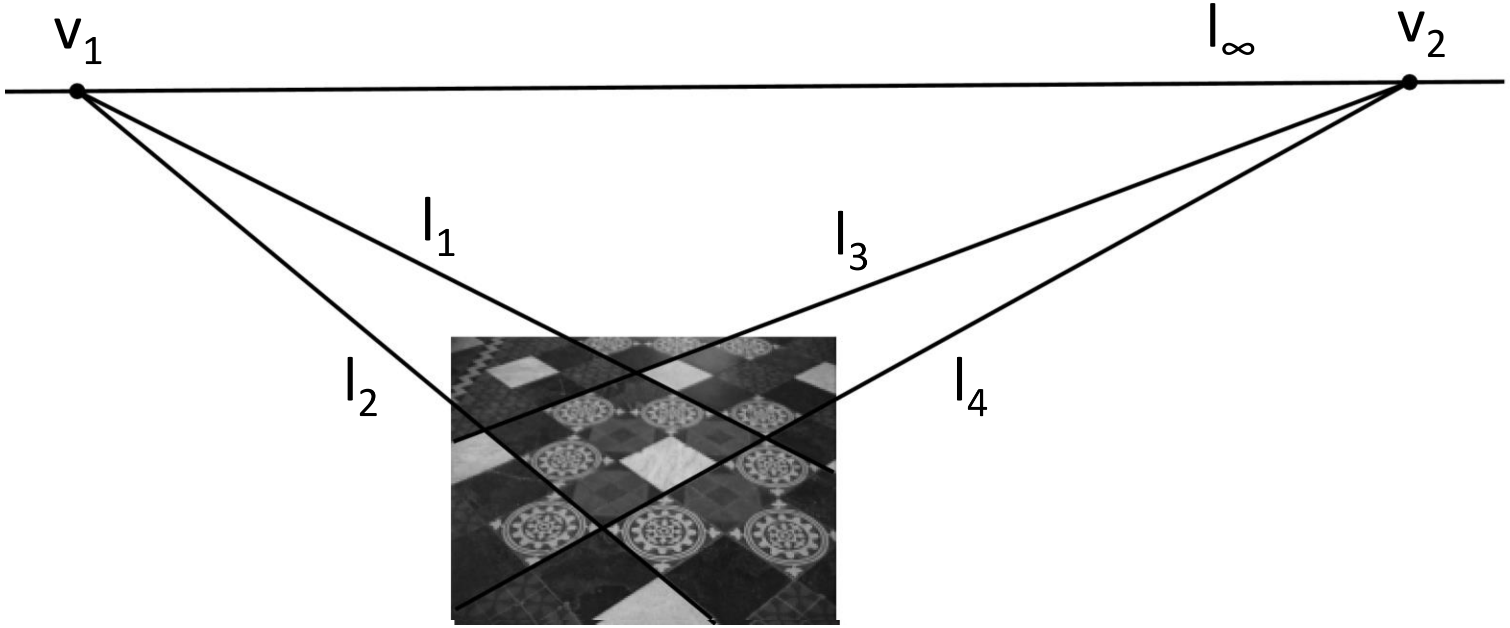
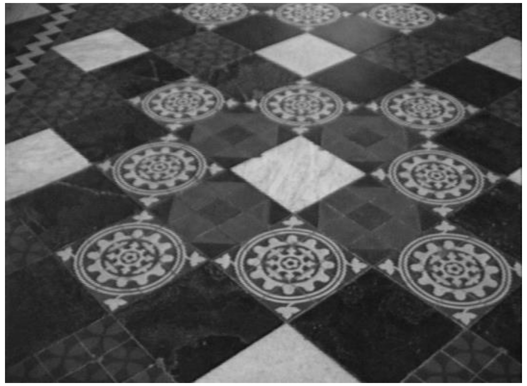
Affine rectification

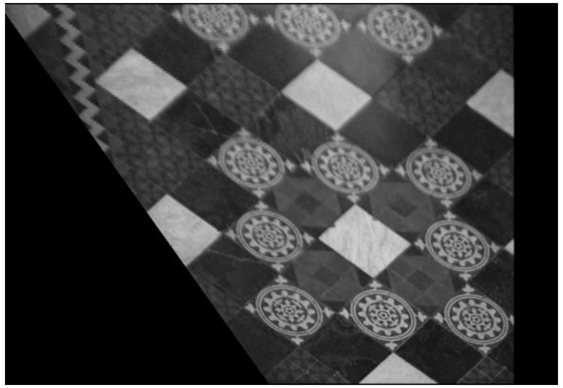


$$v_2 = l_3 \times l_4$$

$$v_1 = l_1 \times l_2$$

$$l_\infty = v_1 \times v_2$$





Point from two lines

$$a_1 x + b_1 y + c_1 = 0$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = 0$$

$$x = x_0 + k_x q$$

$$y = y_0 + k_y q$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix} = \left(a_1 b_2 - a_2 b_1 \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix}$$

$$\begin{bmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} = \left(b_1 c_2 - c_1 b_2 \right) i + \left(c_1 a_2 - a_1 c_2 \right) j + \left(a_1 b_2 - a_2 b_1 \right) k$$

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = - \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = - \frac{1}{\det(A)} \begin{bmatrix} b_2 & -b_1 \\ -a_2 & a_1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{c_c} \begin{bmatrix} b_1 c_2 - c_1 b_2 \\ a_2 c_1 - a_1 c_2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{c_c} \begin{bmatrix} a_c \\ b_c \end{bmatrix}$$

$$\begin{bmatrix} x_h \\ y_h \\ z_h \end{bmatrix} = \begin{bmatrix} a_c \\ b_c \\ c_c \end{bmatrix}$$

Line from two points

$$ax_1 + by_1 + c = 0$$

$$\begin{bmatrix} x_1 & y_1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} i & j & k \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} = \left(y_1 - y_2\right) i + \left(x_2 - x_1\right) j + \left(x_1 y_2 - x_2 y_1\right) k$$

$$\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = -\begin{bmatrix} c \\ c \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = -\frac{1}{c_c} \begin{bmatrix} y_2 & -y_1 \\ -x_2 & x_1 \end{bmatrix} \begin{bmatrix} c \\ c \end{bmatrix}$$

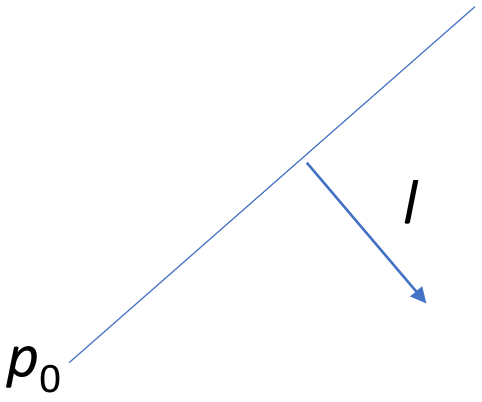
$$\begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{c_c} \begin{bmatrix} a_c \\ b_c \end{bmatrix} c$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a_c \\ b_c \\ c_c \end{bmatrix}$$

Line from parametric equations

$$x = x_0 + k_x q$$
$$y = y_0 + k_y q$$

$$\begin{bmatrix} i & j & k \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix} = (y_1 - y_2)i + (x_2 - x_1)j + (x_1 y_2 - x_2 y_1)k$$



$$\begin{bmatrix} i & j & k \\ x_0 & y_0 & 1 \\ x_0 + k_x & y_0 + k_y & 1 \end{bmatrix} = -k_y i + k_x j + (x_1 y_2 - x_2 y_1)k$$

$$(x_1 y_2 - x_2 y_1) = x_0(y_0 + k_y) - (x_0 + k_x)y_0 = x_0 k_y - k_x y_0$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -k_y \\ k_x \\ x_0 k_y - k_x y_0 \end{bmatrix}$$

$$l = \begin{bmatrix} -k_y & k_x \end{bmatrix}$$
$$p_0 = \begin{bmatrix} x_0 & y_0 \end{bmatrix}$$
$$x_0 k_y - k_x y_0 = -p_0 \bullet l$$

$$l = \begin{bmatrix} -k_y & k_x \end{bmatrix}$$
$$p_0 = \begin{bmatrix} x_0 & y_0 \end{bmatrix}$$
$$x_0 k_y - k_x y_0 = -p_0 \bullet l$$
$$(p - p_0) \bullet l = 0$$

$$(p - p_0) \bullet l = 0$$

parametric equations from line

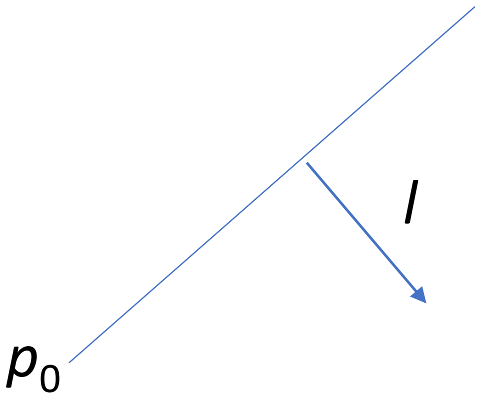
$$ax + by + c = 0$$

$$x = x_0 + k_x q$$
$$y = y_0 + k_y q$$

$$l = \begin{bmatrix} -k_y & k_x \end{bmatrix}$$
$$p_0 = \begin{bmatrix} x_0 & y_0 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} l_x \\ l_y \\ -p_0 \bullet l \end{bmatrix}$$

$$x = x_0 + k_x q$$
$$y = y_0 + k_y q$$

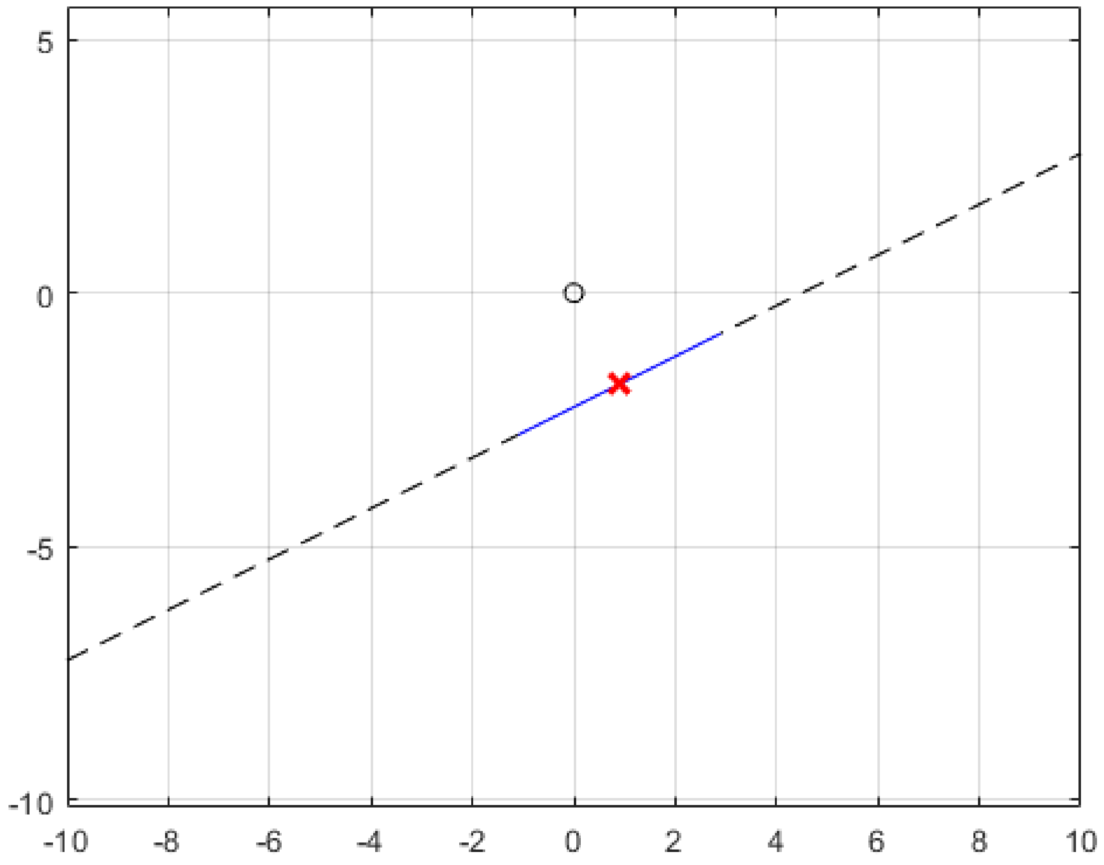


c is distance from line to origin

$$k = \begin{bmatrix} b & -a \end{bmatrix}$$
$$p_0 = \begin{bmatrix} x_0 & y_0 \end{bmatrix}$$

$$p_0 = -\frac{c}{a^2 + b^2}\begin{bmatrix} a & b \end{bmatrix}$$
$$ax_0 + by_0 + c = 0$$
$$-a\frac{ac}{a^2 + b^2} - b\frac{bc}{a^2 + b^2} + c = 0$$
$$0 = 0$$

Verify previous slide




```
a=2;
b=-4;
c=-9;

x = [-10 10];
y = -(c+a*x)/b;
plot(x,y,'k--');
grid on

p0 = -(c/(a^2+b^2))*[a b];
k = [b  -a];
p = [p0-k*0.5; p0+k*0.5];
hold on
plot(p(:,1),p(:,2),'b');
plot(p0(1),p0(2),'rx','MarkerSize',9,'LineWidth',2);
plot(0,0,'ko');
hold off
axis equal
```