

# Digital Signal Processing I ECE 561

## Fall 2003 Test 3

1. Given a digital filter with a pole at  $r = 0.75$  and  $\theta = 30$  degrees from the horizontal axis in the  $z$ -plane.
  - (a) Find the location of a second pole such that the resulting impulse response is real. Find the transfer function ( $b$  and  $a$ ) (5 points)
  - (b) Find the location of the zeros such that the resulting filter is all-pass. Find the transfer function ( $b$  and  $a$ ) (5 points)
2. Given the digital filter described by

$$H(z) = \frac{(1 - 3z^{-1})(1 + 2z^{-1} + 4z^{-2})(2 - z^{-1})}{(1 - 0.5z^{-1})(1 + 0.4z^{-1})}$$

- (a) Generate the pole-zero plot for the filter and tell if this transfer function can be a stable, causal system. (5 points)
  - (b) Plot the group delay as a function of (angular) frequency. (5 points)
  - (c) Transform the filter to a minimum phase filter and show the corresponding pole-zero plot. (5 points)
  - (d) Plot the group delay function of the resulting filter (5 points)
3. Design a filter to remove a 60 Hz spurious component and its first harmonic from a signal sampled at 8000 Hz. Your filter must attenuate these frequencies by a factor of 100, and must attenuate no more than 0.95 for frequencies 1.2 Hz above or below the target frequencies. Show a plot of the magnitude of the response vs. frequency for frequencies from 0 to 200 Hz. (10 points)
4. Given the filter

$$H(z) = -c_4 - c_3z^{-2} - c_2z^{-4} - c_1z^{-6} + c_1z^{-8} + c_2z^{-10} + c_3z^{-12} + c_4z^{-14}$$

where  $c_1 = 0.6121882$ ,  $c_2 = 0.137338$ ,  $c_3 = 0.0324802$ , and  $c_4 = 0.0073304$ . Is this a linear FIR filter? If so, identify its type, and find its amplitude function  $A(\omega)$  in factored form. For example, if the filter were type II, write the amplitude function as

$$A(\omega) = \cos(\omega/2) \sum_{k=0}^K g[k] \cos(k\omega)$$

Plot the magnitude of the filter response over the range  $\omega = -\pi$  to  $\pi$ . (10 points)

5. Design a seventh-order low-pass Chebyshev type II filter with a cutoff frequency of 1500 Hz and attenuated below 30 dB in the stop band. Assume the signal is sampled at 8000 Hz.
- (a) Plot its pole-zero plot. (5 points)
  - (b) Plot its magnitude response. Find the width of the transition band. (5 points)
  - (c) Find the maximum group delay within the passband. (5 points)
  - (d) Generate its State-Space representation. (5 points)
6. Design a FIR filter using a Kaiser window with the same basic specs as above.
- (a) Plot the impulse response. (5 points)
  - (b) Plot the magnitude response. Find the width of the transition band. (5 points)
  - (c) Find the maximum group delay within the passband. (5 points)

7. Design an FIR filter whose frequency response approximates a triangle function of the form

$$A(f) = \begin{cases} 1 - f/f_0 & f \leq f_0 \\ 0 & f > f_0 \end{cases}$$

where  $f_0$  is 3000 Hz and the sampling frequency is 8000 Hz. Plot the magnitude response of your filter. (15 points)