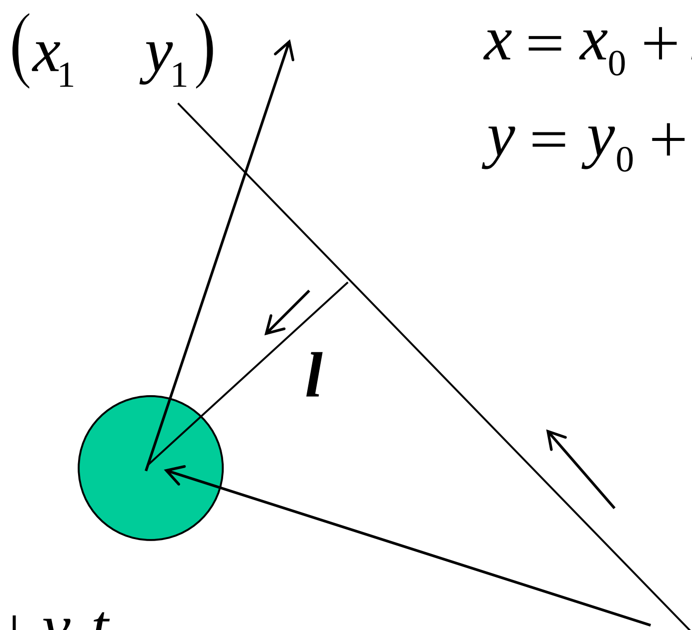
Ball/Line Collision

$$x = x_0 + k_x q$$
$$y = y_0 + k_y q$$



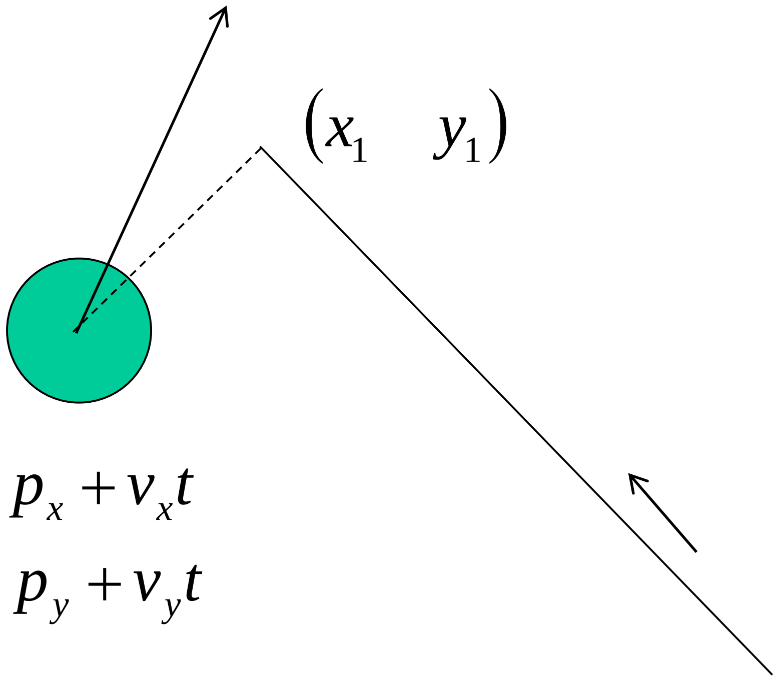
$$l_x = -k_y$$
$$l_y = k_x$$

$$x = p_x + v_x t$$
$$y = p_y + v_y t$$

$$d = \mathbf{p} \cdot \mathbf{l} = -\left(p_x - x_0 + v_x t\right)k_y + \left(p_y - y_0 + v_y t\right)k_x$$
$$d = d_0 + \left(k_x v_y - k_y v_x\right)t$$

$\det\begin{bmatrix} k_x & k_y \\ v_x & v_y \end{bmatrix}$ zero if ball traveling parallel

Ball/endpoint Collision

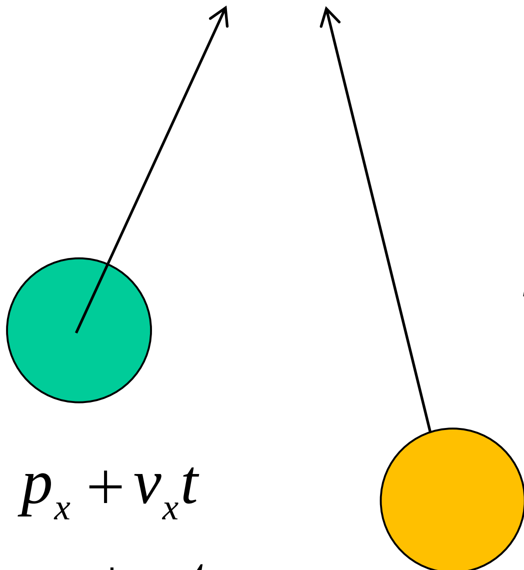


$$x = x_0 + k_x q$$
$$y = y_0 + k_y q$$

$$x = p_x + v_x t$$
$$y = p_y + v_y t$$

$$d^2 = (p_x + v_x t - x_1)^2 + (p_y + v_y t - y_1)^2$$
$$d^2 = d_0^2 + 2\left((p_x - x_1)v_x + (p_y - y_1)v_y\right)t + \left(v_x^2 + v_y^2\right)t^2$$
$$d_0^2 = (p_x - x_1)^2 + (p_y - y_1)^2$$

Ball/Ball Collision



$$x_2 = u_x + k_x t$$
$$y_2 = u_y + k_y t$$

$$x_1 = p_x + v_x t$$
$$y_1 = p_y + v_y t$$

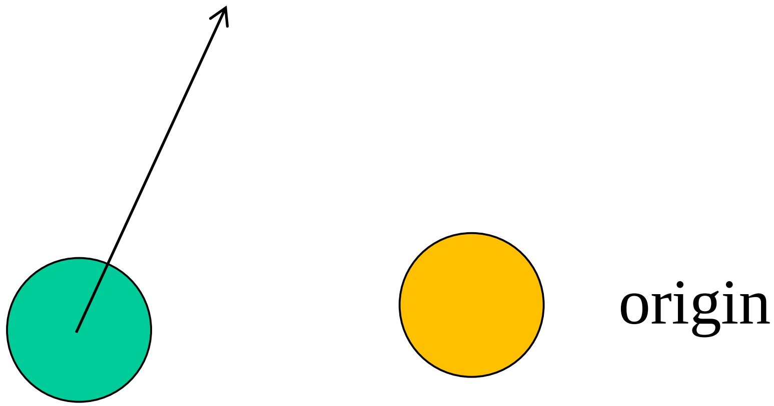
$$d^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2$$

$$d = (u_x + k_x t - p_x - v_x t)$$

$$d^2 = d_0^2 + 2\Big(\left(p_x - x_1\right)v_x + \left(p_y - y_1\right)v_y\Big)t + \left(v_x^2 + v_y^2\right)t^2$$

$$d_0^2 = \left(p_x - x_1\right)^2 + \left(p_y - y_1\right)^2$$

Convert to ball – origin problem



$$x_1 = p_x + v_x t$$

$$y_1 = p_y + v_y t$$

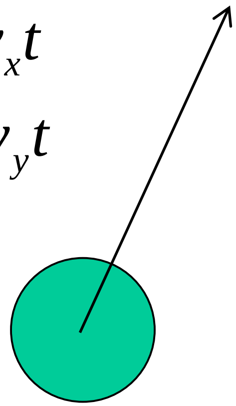
$$d^2 = r_x^2 + r_y^2 = \left(p_x + v_x t\right)^2 + \left(p_y + v_y t\right)^2 =$$

$$d^2 = \left(p_x^2 + p_y^2\right) + 2\left(p_x v_x + p_y v_y\right)t + \left(v_x^2 + v_y^2\right)t^2$$

$$d^2 = C + 2Bt + At^2$$

Convert to ball – origin problem

$$x_1 = p_x + v_x t$$
$$y_1 = p_y + v_y t$$



$$d^2 = r_x^2 + r_y^2 = \left(p_x + v_x t\right)^2 + \left(p_y + v_y t\right)^2 =$$
$$d^2 = \left(p_x^2 + p_y^2\right) + 2\left(p_x v_x + p_y v_y\right)t + \left(v_x^2 + v_y^2\right)t^2$$
$$d^2 = C + 2Bt + At^2$$

$$d_0 = \text{radius}_1 + \text{radius}_2$$

If $C < d_0^2$ balls are already intersecting (at $t = 0$)

If $A = 0$, there is no relative velocity

If $B > 0$, the balls are moving apart

$$At^2 + 2Bt + C = 0$$
$$t = \frac{-B \pm \sqrt{R}}{A}$$
$$R = B^2 - AC$$