Convolutions

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$$\int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau \tag{1}$$

1 rect $*$ rect

$$\mathrm{rect}(t/a) * \mathrm{rect}(t/b) \tag{2}$$

where $a < b$.

Define $t_{\max} = (a+b)/2$ and $t_{\min} = (b-a)/2$.

Region 1 extends from $t = -t_{\max}$ to $t = -t_{\min}$ (partial overlap).

$$g(x) = \int_{-a/2}^{t+b/2} d\tau = t + b/2 + a/2 = t + t_{\max}. \tag{3}$$

Region 2 extends from $t \geq -t_{\min}$ to $t \leq t_{\min}$ (full overlap)

$$g(x) = a \tag{4}$$

Region 3 extends from $t = t_{\min}$ to $t = t_{\max}$ (partial overlap).

$$g(x) = \int_{t-b/2}^{a/2} d\tau = a/2 - (t - b/2) = t_{\max} - t \tag{5}$$

2 $e^{-at}u(t) * u(t)$

$$\int_0^t e^{-a\tau} d\tau = \frac{1 - e^{-at}}{a} \tag{6}$$

3 $e^{-at}u(t) * e^{-bt}u(t)$

$$\int_0^t e^{-a\tau}e^{b\tau-bt}d\tau = e^{-bt}\int_0^t e^{-(a-b)\tau}e^{b\tau}d\tau = \frac{e^{-bt}-e^{-at}}{a-b} \tag{7}$$

If $a = b$, we can simplify the result to te^{-at}.
If $b = 0$, the results simplify to the previous section.