

# Signals and Systems ECE 202

## Assignment 4

1. Sketch the following CT function

$$\int_{-\infty}^t [\delta(\lambda + 1) - 2\delta(\lambda) + \delta(\lambda - 1)] d\lambda \quad (1)$$

2. Use a change of variable to show that

$$\delta(a(t - t_0)) = \frac{1}{|a|} \delta(t - t_0) \quad (2)$$

3. Using the result of the previous problem, show that

(a)

$$\text{comb}(ax) = \frac{1}{|a|} \sum_{n=-\infty}^{\infty} \delta\left(x - \frac{n}{a}\right) \quad (3)$$

(b) The average value of  $\text{comb}(ax)$  is one, independent of the value of  $a$ .

(c) A comb function of the form  $(1/a)\text{comb}(t/a)$  is a sequence of *unit* impulses spaced  $a$  units apart.

(d) Even though  $\delta(at) = (1/|a|)\delta(t)$ ,  $\text{comb}(at) \neq (1/|a|) \text{comb}(t)$ .

4. What is the numerical value of each of the following integrals?

$$\int_{-\infty}^{\infty} \text{comb}(t) \cos(48\pi t) dt \quad (4)$$

$$\int_{-\infty}^{\infty} \text{comb}(t) \sin(2\pi t) dt \quad (5)$$

$$\int_0^{20} \text{comb}\left(\frac{t-2}{4}\right) \text{rect}(11t) dt \quad (6)$$

$$\int_{-2}^2 \text{comb}(t) \text{sinc}(t) dt \quad (7)$$

5. Sketch these convolutions

(a)  $g(t) = 3 \cos(10\pi t) * \delta\left(t + \frac{1}{10}\right)$

(b)  $g(t) = \text{tri}(2t) * \text{comb}(t)$

(c)  $g(t) = \text{sinc}(4t) * \frac{1}{2} \text{comb}\left(\frac{t}{2}\right)$